

# DIRECT PARTICLE INTERACTION AS THE ORIGIN OF THE QUANTAL BEHAVIOURS

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# DIRECT PARTICLE INTERACTION AS THE ORIGIN OF THE QUANTAL BEHAVIOURS

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## ABSTRACT

*It is argued that the quantal behaviours may be understood in the framework of direct particle interactions. A specific example is introduced. The assumed potential predicts that at sufficiently large distances quantal behaviours arise, while at very large distances gravitational-like forces are present. The latter is true provided all particles have internal structures.*

## 1 Direct Particle Interaction Versus Field Theory

In formulating his ideas and those of former physicists about motion and gravitation, Newton made two fundamental assumptions:

- (a)– He postulated the presence of *absolute space and time* — physical objects which act on the particles, but are not acted upon by them.
- (b)– He assumed that absolute space is isotropic and homogeneous and that

absolute time is also homogeneous. In other words, he supposed that physics is invariant under Galileo's transformations.

An important result of these assumptions is that the particles interact at a distance, i.e. information propagates with infinite velocity[1]. Theories containing *action-at-a-distance* (AAAD) interactions are examples of *direct particle interaction* (DPI) theories.

Some people felt uncomfortable with the above assumptions. Philosophers like Berkeley, Leibnitz, Mach and others, argued that the only physically meaningful thing for moving particles is the *relative motion* and thus it is difficult and unnecessary to believe in absolute space and time. It is possible to show that such a relational physics must be invariant under Leibnitz transformations[2]:

$$\begin{cases} \vec{x} \rightarrow \vec{A}(t) \cdot \vec{x} + \vec{B}(t) \\ t \rightarrow C(t) \end{cases} \quad (1)$$

where  $\vec{A}(t)$  is an orthogonal matrix and  $\vec{B}(t)$  and  $C(t)$  are arbitrary functions of time. Barbour et.al.[2-5] have constructed lagrangians which are Leibnitz-invariant and shown that locally one arrives at our standard physics. They are also able to relate local parameters to cosmological ones in their theory.

The apparent contradiction between Maxwell's electromagnetic theory and Newton's theory of motion, led some people to doubt the absoluteness of space

and time and to the postulation of absolute space-time. In this way the special theory of relativity and Lorentz transformations were born. Later when Einstein tried to bring gravitation in this framework, he was forced to throw out the absoluteness of space-time but not its existence. An important result of relativity theory is that the velocity of particles as well as the information propagation velocity cannot exceed a universal value — the velocity of light. Accordingly, there are two possible ways of describing the interaction between particles. First, one can introduce the *field* concept, a physical object which propagates with a finite velocity which is less than or equal to that of light. All information between particles is carried by the field. Second, it is possible to look at the situation through the glasses of DPI. Particles interact with each other directly but not instantaneously. In the case of electromagnetism, where the velocity of propagation of information is equal to that of light, we have *action-at-a-zero-proper-distance* (AAAZPD). The first suggestion is what actually is used in Maxwell's theory of electromagnetism. Schwarzschild, Tetrode, Fokker, Wheeler, Feynman, Hoyle, and Narlikar[6-18] developed a DPI theory for electromagnetism which produces all of the results of Maxwell's theory and in addition predicts two important things — the self force and the existence of only the retarded solutions.

On the other hand, quantum mechanics brought with itself idealism, inde-

terminism and nonlocality. The first two have been overcome in the beautiful theory of Bohm[19-25]. He showed how one can remain faithful to realism and determinism as in classical physics and at the same time be able to reproduce all of the results of quantum mechanics, via introducing a *quantum potential* (QP) in Newton's equation of motion. In this way, quantum phenomena are nothing but physical situations in which a new force, the quantum force, derivable from the QP:

$$Q = -\frac{\hbar^2}{2m} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} \quad (2)$$

is present. In the above relation  $\rho(\vec{x}, t)$  is the density of an ensemble of particles. The QP has the peculiar property that it is not dependent on the magnitude of density, it is a function of the shape of the square root of it. The nonlocality of quantum mechanics (i.e. the presence of AAAD) which can be seen both through the QP[25,26] and in Bell's theorem[27], has apparently been proved experimentally[28]. Therefore one is forced to set some limitations on the validity domain of the relativity theory. The QP always acts via AAAD. It cannot be formulated either as an AAZPD or as a field theory. Although some works have been done, to make Bohm's theory consistent with relativity[29-31], none of them are acceptable, either because of theoretical problems or because of the lack of agreement with experiments.

In addition to this apparent contradiction between relativity and quantum theories, there is yet another problem. When one combines these theories and applies it to Maxwell's theory, one gets some amazing results. In Maxwell's theory the interaction between charged particles is transported by the electromagnetic field, while in quantum electrodynamics, this interaction is mediated by *particle-like-states* called *photons*. It is always stated that photons travel at light's speed and thus there is no room for mysterious AAAD. But, we must note that first of all, the virtual photons are not on the mass shell, so they may have any velocity from zero to infinity. Although when one sums over all possible paths, the result is Lorentz-invariant, this seems to be in contradiction with the spirit of relativity theory. Second, although quantum field theory removes the need for AAAD, it leads to infinities. Investigation of some of these infinities (by the point splitting renormalization method, say) shows that they are the results of interaction of photons and charged particles at a point (*action-at-a-point* or AAAP). To avoid them, one must let the interaction takes place at a distance! In summary one can choose either AAAD or AAAP, but the latter leads to infinities.

Summing up our discussion, one is forced to accept that the correct physical theory must be relational and containing DPI. We ruled out Galileo and Lorentz transformations and chose Leibnitz transformations because of two

facts. First, we know that they are only of limited validity, and, as Barbour et.al.[2-5] have shown, they are local approximations of Leibnitz transformations. Second, a relational physics rules out the unphysically existing self-dependent space-time.

In the following, a specific and appropriate DPI is suggested, using the general properties of DPIs. This typical DPI is founded on a trivial, tautological postulate. It contains two scale factors, a short scale  $\alpha_s$  and a large scale  $\alpha_\ell$ . It will be shown that at distances larger than  $\alpha_s$ , this prototype DPI is equal to the Bohm's QP plus some small corrections. Therefore it provides a framework for understanding the mysterious quantal behaviours in terms of instantaneous interaction between different particles of the ensemble. As it is wellknown[25], the QP plus the natural constraint on density to obey the continuity equation, leads to the schrodinger equation. So in fact we shall derive the quantum theory from DPI. An important property of our prototype DPI is that its small corrections to the QP magnify the internal structure of any particle at large distances. They lead to gravitational-like forces. Therefore it is suggested that DPI theories are suitable for unifying gravity and quantum mechanics (which from Bohm's point of view is nothing but a fifth force). A good DPI theory must unify at the same time all of the five forces (gravity, electromagnetism, weak, strong and quantum forces).

## 2 A Typical DPI

As it was discussed in the previous section, many areas of physics, including quantum mechanics and Newtonian gravity, are understandable in terms of AAAD, i.e. nonlocality. It is also argued that DPI theories seem to be a natural framework for describing nonlocal phenomena and thus for unifying different parts of physics. In this section we shall develop a typical DPI, and later, in the forthcoming sections, we show that under certain conditions it reduces to the QP or to the Newtonian gravity.

To begin with, let us stress a trivial property of DPIs. It is clear that any DPI, highly depends upon the configuration of particles, i.e. on their relative position. Therefore the first task in constructing any DPI is to ensure that each particle is at its correct position, i.e. the position derived from the equation of motion. Accordingly we postulate the following tautological statement:

**Postulate:** *Each particle is at its own location.*

It seems unbelievable that this postulate can lead to any physical conse-



quences, but as we shall see, it is essential in obtaining the QP.

Now let us formulate this postulate. Consider a system of  $N$  identical particles, each one located at  $\vec{a}_i(t)$ ,  $i = 1 \cdots N$ . One can imagine that this pattern of particles is made by bringing particles in one by one, and locating them at their correct position. In order to ensure that each particle is at its right position, the DPI potential should contain a factor, which is infinitely large when some particle is at incorrect position and is finite otherwise. This can be achieved for each particle, if we make use of the Dirac delta function, in the form  $1/\delta(\vec{x} - \vec{a}_i(t))$ . Since we assume that all particles are identical, each particle may be put at any of  $\vec{a}_i(t)$ 's. So the corresponding factor in the DPI potential is  $1/\sum_{i=1}^N \delta(\vec{x} - \vec{a}_i(t))$ . This is zero when  $\vec{x}$  is equal to some legal position and is infinite elsewhere.

Apart from this factor, the DPI potential may contain a factor (obviously relational) representing relative configuration of particles. To have a definite model, we assume two kinds of interactions: a short range interaction and a long range one with ranges  $\alpha_s$  and  $\alpha_\ell$  respectively. In addition, we assume these interactions be exponential. Therefore, as a typical DPI potential, we consider the following one:

$$U(\vec{x}, t) = \frac{U_0}{\sum_{i=1}^N \delta(\vec{x} - \vec{a}_i(t))} \left\{ \sum_{j=1}^N \exp[-(\vec{x} - \vec{a}_j(t))^2/\alpha_s^2] \right\} \left\{ \sum_{k=1}^N \exp[-(\vec{x} - \vec{a}_k(t))^2/\alpha_\ell^2] \right\} \quad (3)$$

where  $U_0$  is some constant. We shall work with this DPI potential throughout this paper. Two notes must be remarked here. First, the first factor is equal to  $1/\rho(\vec{x}, t)$ . This says the following: *particles like to go where particles are present*. Second, the exponential form is not necessary. In fact, if these terms fall faster than  $1/x^2$  all of the forthcoming results can be obtained. We choose this form for simplicity. In other parts of this work we shall show that the QP and the Newtonian gravitational potential are derivable from this prototype DPI potential.

### 3 QP As A Result Of DPI

In this section, we use our prototype DPI to derive the Bohm's QP. Suppose we are dealing with particle separations larger than  $\alpha_s$ . Multiply  $U(\vec{x}, t)$  by the identity factor  $1 = (\pi\alpha_s^2)^{3/2}/(\pi\alpha_s^2)^{3/2}$ , and use the identity:

$$\int d^3y \exp\{-[\vec{y} + \beta(\vec{x} - \vec{a}_k(t))]^2/\alpha_s^2 + y^2/\gamma^2 - y^2/\gamma^2\} = (\pi\alpha_s^2)^{3/2} \quad (4)$$

for arbitrary  $\beta$  and  $\gamma$ . By choosing  $\beta$  and  $\gamma$  in the form:

$$\beta = \frac{1}{2} \left[ 1 + \sqrt{1 - 4\alpha_s^2/\alpha_\ell^2} \right] \quad (5)$$

$$\gamma = \frac{\sqrt{2}\alpha_s}{\left[ 1 - \sqrt{1 - 4\alpha_s^2/\alpha_\ell^2} \right]^{1/2}} \quad (6)$$

and after a little algebra and noting that for small  $\epsilon$  (or in other words for  $(\vec{x} - \vec{a}_k(t))^2 \gg \epsilon^2$ ) we have the following representation for the square root of Dirac's delta function:<sup>1</sup>

$$\left(\frac{1}{\pi\epsilon^2}\right)^{3/2} \exp[-(\vec{x} - \vec{a}_k(t))^2/\epsilon^2] \simeq \delta(\vec{x} - \vec{a}_k(t)) \quad (7)$$

with  $2\epsilon^2 = \alpha_s^2$  and using the following fact:

$$\sum_{k=1}^N \sqrt{\delta(\vec{x} - \vec{a}_k(t))} = \sqrt{\sum_{k=1}^N \delta(\vec{x} - \vec{a}_k(t))} \quad (8)$$

which can be proved by using the step function representation of Dirac's delta function, and using the definition of the density of particles:

$$\rho(\vec{x}, t) = \sum_{k=1}^N \delta(\vec{x} - \vec{a}_k(t)) \quad (9)$$

one can easily show that the DPI potential can be written as:

$$U(\vec{x}, t) \simeq U_0(4\beta)^{-3/4} \int d^3y \sqrt{\frac{\rho(\vec{x} + \vec{y}, t)}{\rho(\vec{x}, t)}} \exp\left[-\frac{y^2}{\gamma^2}\right] \quad (10)$$

This is an equivalent form of equation (3) for our typical DPI, written in terms of the density of the ensemble. But it must be noted that equations (10) and

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<sup>1</sup>Note that we have used the parameter  $\alpha_s$  as the small parameter of the representation of Dirac's delta function. It may seem that it is an arbitrary choice, but it can be seen that if one chooses another small parameter, the result is of the same form as in (21) with different coefficients. The above choice is the most economical one.

(3) are the same, only for  $(\vec{x} - \vec{a}_k)^2 \gg \alpha_s^2$ . Thus we assume that the correct potential, both for small and large separations is (3).

Our aim is now, to show the relation between (3) or (10) and Bohm's QP. In order to do this, we express  $\sqrt{\rho(\vec{x} + \vec{y}, t)}$  in terms of (7)–(9). The integral is then Gaussian and can be carried out:

$$U(\vec{x}, t) \simeq U_0 \left( \frac{1}{4\pi\epsilon^2\beta} \right)^{3/4} \left( \frac{\pi}{1/2\epsilon^2 + 1/\gamma^2} \right)^{3/2} \frac{\sum_{k=1}^N \exp \left[ -\frac{(\vec{x} - \vec{a}_k(t))^2}{2\epsilon^2 + \gamma^2} \right]}{\sqrt{\rho(\vec{x}, t)}} \quad (11)$$

Note that for  $\alpha_l \gg \alpha_s$  this is proportional to:

$$\frac{\sum_{k=1}^N \exp [-(\vec{x} - \vec{a}_k(t))^2/\alpha_l^2]}{\sum_{k=1}^N \exp [-(\vec{x} - \vec{a}_k(t))^2/2\epsilon^2]}$$

This is a form which can be obtained directly from equation (3) by using the above Gaussian representation of Dirac's delta function.

Equation (11) is a form for DPI that contains both the density and the particles' position. Relation to Bohm's QP would be appear if the DPI is written in terms of the density, only. In order to arrive at this aim, we use Backer–Hausdrof lemma:

$$e^{-G} A e^G = A - [G, A] + \frac{1}{2}[G, [G, A]] + \dots \quad (12)$$

and set  $G = G(\vec{x})$  and  $A = \vec{\nabla}$ . Nothing that

$$[G, \vec{\nabla}] = -\vec{\nabla} G, \quad [G, [G, \vec{\nabla}]] = 0 \quad etc. \quad (13)$$

we have

$$e^{-G}\vec{\nabla}e^G = \vec{\nabla} + \vec{\nabla}G \quad (14)$$

This is an operator identity. Now suppose that it acts on the unity:

$$\left(e^{-G}\vec{\nabla}e^G\right)1 = \left(\vec{\nabla}G\right)1 \quad (15)$$

If one uses this relation for the following operator:

$$e^{\omega\nabla^2} \equiv 1 + \omega\nabla^2 + \dots \quad (16)$$

one has:

$$\left(e^{-G}e^{\omega\nabla^2}e^G\right)1 = e^{\omega|\vec{\nabla}G|^2} \quad (17)$$

In this relation we choose:

$$G = -\frac{(\vec{x} - \vec{a}_k(t))^2}{2\epsilon^2} \quad (18)$$

So:

$$e^{\omega\nabla^2}e^{-(\vec{x}-\vec{a}_k(t))^2/2\epsilon^2} = e^{(-1/2\epsilon^2+\omega/\epsilon^4)(\vec{x}-\vec{a}_k(t))^2} \quad (19)$$

If one sets:

$$\omega = \frac{1}{2} \frac{\epsilon^2\gamma^2}{2\epsilon^2 + \gamma^2} \quad (20)$$

the relation (11) can be simplified as:

$$U(\vec{x}, t) \simeq U_0 \left( \frac{1}{4\pi\epsilon^2\beta} \right)^{3/4} \left( \frac{\pi}{1/2\epsilon^2 + 1/\gamma^2} \right)^{3/2} \frac{e^{\omega\nabla^2} \sum_{k=1}^N \exp \left[ -\frac{(\vec{x}-\vec{a}_k(t))^2}{2\epsilon^2} \right]}{\sqrt{\rho(\vec{x}, t)}}$$

$$\begin{aligned}
&= U_0 \left( \frac{1}{4\pi\epsilon^2\beta} \right)^{3/4} \left( \frac{\pi}{1/2\epsilon^2 + 1/\gamma^2} \right)^{3/2} \frac{e^{\omega\nabla^2} \sqrt{\rho}}{\sqrt{\rho}} \\
&= U_0 \left( \frac{1}{4\pi\epsilon^2\beta} \right)^{3/4} \left( \frac{\pi}{1/2\epsilon^2 + 1/\gamma^2} \right)^{3/2} \frac{1}{\sqrt{\rho}} (1 + \omega\nabla^2 \sqrt{\rho} + \dots)
\end{aligned} \tag{21}$$

But this is just Bohm's QP corrected by small terms! Thus, conclusion is that at separations larger than  $\alpha_s$ , our DPI leads to Bohm's QP with some corrections. Note that the approximate nature of (21) is due to (7).

At this point it is worthwhile to note that our DPI potential in form (3) seems to be large when  $\rho$  is small, but this property is not apparant in (21). A glance at the derivation of (21) from (3) shows that the exponential terms in (3) are related to  $\rho$ , and thus in taking the limit of small  $\rho$ , their role must be considered. In fact the case of small densities needs some caution. (See e.g. [25])

## 4 Observations

We have seen that Bohm's QP, may be viewed as a result of a DPI. Here is some points:

(a)– In the previous section, we see that for a system of  $N$  similar particles,

our prototype DPI led to the QP. In our derivations an essential assumption was made. It was assumed that the density function  $\rho(\vec{x}, t)$  is a differentiable function of  $\vec{x}$ . This is true only for large  $N$ . Therefore the prototype DPI leads to the QP for a large ensemble of particles.

(b)– Although the QP may be derived for an ensemble of similar particles, it is also the correct potential for a particle observed a large number of times. The term representing our tautological postulate must be interpreted as follows. The DPI potential has to be finite when the particle is at any allowed position and infinite elsewhere. The exponential terms have similar interpretations.

(c)– The above one-particle derivation may be generalized to the many-particle case. Consider  $N$  particles of kind 1,  $N$  particles of kind 2,  $\dots$ , and  $N$  particles of kind  $m$ . Then the DPI potential must be written as:

$$U(\vec{x}_1, \dots, \vec{x}_m, t) = \frac{U_0}{\sum_{i=1}^N \prod_{l=1}^m \delta(\vec{x}_l - \vec{a}_i^{(l)}(t))} \times \left\{ \sum_{j=1}^N \prod_{l=1}^m \exp[-(\vec{x}_l - \vec{a}_j^{(l)}(t))^2 / \alpha_s] \right\} \left\{ \sum_{k=1}^N \prod_{l=1}^m \exp[-(\vec{x}_l - \vec{a}_k^{(l)}(t))^2 / \alpha_\ell] \right\} \quad (22)$$

where  $\vec{a}_i^{(l)}(t)$  represents a legal position for a particle of kind  $l$ . In a manner very similar to that of the one-particle case, one can show that the DPI potential

can be written as:

$$U(\vec{x}_1, \dots, \vec{x}_m, t) \simeq U_0(4\beta)^{-3m/4} \left( \frac{\pi}{1/2\epsilon^2 + 1/\gamma^2} \right)^{3m/4} \frac{1}{\sqrt{\rho(\vec{x}_1, \dots, \vec{x}_m, t)}} \\ \times \exp \left\{ \omega[\nabla_1^2 + \dots + \nabla_m^2] \right\} \sqrt{\rho(\vec{x}_1, \dots, \vec{x}_m, t)} \quad (23)$$

for separations larger than  $\alpha_s$  and for large  $N$ . This is just Bohm's expression for the QP of a many-particle system, corrected by small terms. Now, although we have derived the above expression for an ensemble of similar many-particle systems, it is also true for any many-particle system.

(d)– As it was seen in the previous section, our prototype DPI potential is equivalent to Bohm's QP with some small corrections. Now we show that this correction terms have the property of magnifying the small scale structure of matter to large distances. Since in the expression (21), derivatives of very high degree exist, very far points are connected. To see this, suppose we discretize the space by unit  $\epsilon$ , and consider  $\rho(\vec{x}, t)$  to be spherically symmetric.

Then we have:

$$\nabla^2 \sqrt{\rho} \simeq \frac{\sqrt{\rho(r - 2\epsilon)}}{\epsilon^2} + \text{other terms.} \quad (24)$$

So the DPI potential may be written as:

$$U(\vec{x}, t) \simeq U_0(4\beta)^{-3/4} \left( \frac{\pi}{1/2\epsilon^2 + 1/\gamma^2} \right)^{3/4} \frac{1}{\sqrt{\rho(r)}}$$



$$\times \sum_{n=0}^{M/2-1} \frac{1}{n!} \omega^n \frac{1}{\epsilon^{2n}} \sqrt{\rho(r - 2n\epsilon)} + \text{other terms.} \quad (25)$$

where

$$M = \frac{r}{\epsilon} \quad (26)$$

If we choose

$$\epsilon = \sqrt{\omega} \simeq \frac{\alpha_s}{2} \quad (27)$$

we obtain:

$$U(\vec{x}, t) \simeq U_0 (4\beta)^{-3/4} \left( \frac{\pi}{1/2\epsilon^2 + 1/\gamma^2} \right)^{3/4} \frac{1}{\sqrt{\rho(r)}} \sum_{n=0}^{M/2-1} \frac{1}{n!} \sqrt{\rho((1 - 2n/N)r)} + \text{other terms.} \quad (28)$$

Note that this expression is acceptable only for

$$r \gg \epsilon \sim \alpha_s \quad (29)$$

As it is seen in this relation, the DPI potential relates any point, to the density at a distant location.

As a model, suppose that the universe is made of a uniform distribution of matter with density  $\rho_0$  and a particle with very fine internal structure like below:

$$\rho(r) = \zeta^2 \left[ 1 - e^{-\xi/r} \right]^2 \quad (30)$$

This is a sharp function of  $r$  provided  $\xi$  is very small. The DPI potential is

now:

$$U(\vec{x}, t) \simeq \frac{U_0(4\beta)^{-3/4} \left( \frac{\pi}{1/2\epsilon^2 + 1/\gamma^2} \right)^{3/4}}{\sqrt{\rho(r)}} \zeta \xi \left( \sum_{n=0}^{M/2-1} \frac{1}{n!(1-2n/N)} \right) \frac{1}{r} + \text{other terms.} \quad (31)$$

This is just the Newton's law of gravitation plus some corrections. Note that it has the correct sign and that there is a relation between the Plank's constant, Newton's constant of gravitation, the density of universe and the parameters of the internal structure of particles.

In summary, if we assume that all particles have internal structures below the  $\alpha_s$  scale, at large distances ( $r \gg \alpha_s$ ) one sees some gravitation-like forces!

(e)– To complete our discussion, we must consider the kinetic terms and study the dynamics of the system. Barbour et. al.[2-5] have shown that Leibnitz-invariant lagrangians are of the form of the product of a kinetic term  $\mathcal{K}$  and a potential term  $\mathcal{P}$ :

$$\mathcal{L} = \mathcal{K}\mathcal{P} \quad (32)$$

In our case, for a one-particle system, the potential term is

$$\mathcal{P} = \sum_{i=1}^N U(\vec{a}_i(t)) \quad (33)$$

while we have chosen the kinetic term to be

$$\mathcal{K} = \left( \sum_{i=1}^N \left[ \frac{d}{dt} | \vec{a}_i(t) - \vec{a}_j(t) | \right]^2 \right)^{1/2} \quad (34)$$

Barbour and others[2] have shown that the exponent one-half is necessary for the action be Leibnitz-invariant. In accordance with their work, one is able to relate the local physics to cosmology. In this way the coupling constant of the quantum force, i.e.  $-\hbar^2/2m$  is related to the cosmological parameters like the radius of the universe, its the expansion velocity, its density and so on. Therefore  $\hbar$  may be a function of time depending on our choice of cosmological model.

(f)– As a cosmological model, Consider the universe made of a shell of radius  $R$ , moving radially with velocity  $\dot{R}$ , as well as  $N$  particles located at  $\vec{a}_i(t)$ . Then the kinetic term can be written as sum over shell-shell points plus sum over shell-particles plus sum over particle-particle. The result is (see [2] for a similar calculation):

$$\mathcal{K} = \text{constant} \left[ 1 + \frac{1}{30\dot{R}^2} \sum_{i=1}^N \left( \frac{d\vec{a}_i}{dt} \right)^2 + \dots \right] \quad (35)$$

So the lagrangian is approximately:

$$\mathcal{L} \simeq \text{constant} \left\{ \frac{1}{2}m \sum_{i=1}^N \left( \frac{d\vec{a}_i}{dt} \right)^2 - \frac{\hbar^2}{2m} \sum_{i=1}^N \left( \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} \right)_{\vec{x}=\vec{a}_i} - Gm \sum_{i=1}^N \frac{1}{|\vec{a}_i - \vec{a}_j|} + \dots \right\} \quad (36)$$

where we have:

$$\frac{\hbar(t)}{\hbar(t_0)} = \frac{\dot{R}(t)}{\dot{R}(t_0)} \quad (37)$$

and:

$$\frac{G(t)}{G(t_0)} = \sqrt{\frac{\rho_{universe}(t_0)}{\rho_{universe}(t)}} \quad (38)$$

## 5 Conclusion

It is shown that the DPI is a natural framework for quantal phenomena. As our prototype DPI potential shows, one is able to describe both the QP and the gravity as different aspects of a single interaction, provided that all particles have internal structure. Thus we hope that different parts of physics may be different aspects of a specific DPI, although the construction of such a DPI model needs a large amount of work.

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